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Relationships between solar radiation and solar altitude, at Cambridge, over the growing season

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1. Introduction

It is perhaps worthwhile to remind ourselves of a statement by Stagg over 20 years ago

"As these records (from Kew Observatory) form one of the few sources of information on solar radiation in the British Isles, and because of their interest to meteorologists as well as to architects, specialists in building research, heating and fuel engineers and agriculture and fishery authorities, it is desirable to summarise the data."

Such summaries as are available for radiation data deal generally with the radiation received on a horizontal surface (at the bottom of the atmosphere ie ground level).

If one is concerned with the radiation intercepted by a surface as a function of its aspect (orientation) and tilt (elevation from the horizontal), as the potential users of information on radiation loads must be, then conventional summaries of radiation data are of little use, unless supplemented by relationships between solar radiation and solar altitude.

2. Basic expressions

We may write an expression for the radiation falling on a plane at the earth's surface, the plane being at an angle (E) to the horizontal, as

$$S = I_{\delta} \cos\beta \cdot (n/N) \quad \text{- Direct component}$$

$$+ \left[\overline{D}_{\delta A} \cdot (n/N) + f'(n/N)\,(I_{\delta} \sin\delta + D_{\delta A}) \right] \cos^2 E/2 \quad \text{- Diffuse sky component}$$

$$+ \left[\overline{I}_{\delta} \sin\delta + D_{\delta A})(n/N) + f'(n'/N')(I_{\delta} \sin\delta + D_{\delta A}) \right] A' \,(1-\cos^2 E/2) \quad \text{- Reflected ground component}$$

Where S = total (global) solar radiation

I_γ = direct solar radiation at the earth's surface on a plane normal to the solar beam.

β = angle of incidence of the direct solar beam onto the plane of interest
(the angle between the solar beam and the normal to the plane)

E = tilt of the plane to the horizontal.

A = general albedo of the region

A' = albedo immediately in front of the receiving plane

γ = angle of elevation of the sun (measured from a horizontal surface)

n/N = relative duration of sunshine
(the ratio of the actual hours of bright sunlight to the possible hours of bright sunlight ie day length)

$D_{\gamma A}$ = scattered (diffuse) radiation from a clear sky

f'(n/N) = a constant of proportionality, when (n/N) is determined from monthly mean values of cloudiness, and which depends on the value of (n/N).

Geometry indicates that

$$\sin\gamma = \sin\phi \sin\delta + \cos t \cos\phi \cos\delta \cos t$$

where γ = solar elevation

ϕ = latitude of the plane

δ = solar declination (assumed constant on any one day)

t = hour angle of the sun, as seen from a horizontal surface
It is measured from local apparent noon and counted positive after local apparent noon and negative before local apparent noon.

3. The purpose of the study

The object was to develop relationships between I_γ, D_γ and γ, such that when the expression (S) was evaluated for a typical day in each month (through the summation of hourly values), the long period average, daily mean values of Direct and Diffuse radiation on a horizontal surface were given.

4. Procedure

April is considered as the typical month.

For the 10 or more Aprils for which radiation data was available on a daily (and hourly) basis, the day in each month which showed the highest daily value of total radiation on a horizontal surface was identified. These days were placed in ranking order, and the first five days were used to build up a picture of the variation through the day on occasions when total radiation was extreme. Symmetry about noon was assumed and extreme. hourly values of total radiation on a horizontal surface were found.

The assumption was made that on this composite day the ratio $(n/N) = 1$, that direct radiation was likely to be an order of magnitude greater than diffuse and that accordingly, diffuse radiation could be ignored.

The amount of direct radiation falling on a horizontal surface was found for this (extreme) composite day. This daily sum was multiplied by the long term average value of (n/N) appropriate to April at Cambridge and the result compared with the long term, daily mean value of direct radiation on a horizontal surface.

The hourly values of the (extreme) composite day for direct radiation were now scaled down uniformly to give the hourly values of direct radiation appropriate to the average day.

For a horizontal surface, $I_\beta \cos\beta = I_\gamma \sin\gamma$ and at this stage, we have hourly values of the term $I_\gamma \sin\gamma$, which when summed for the day, give the observed, long period, daily mean value of Direct radiation on a horizontal surface.

With the usual assumption that radiation evaluated at h_1 + 30 minutes provides a suitable mean for radiation between the hours h_1 and h_2 we can now assign values to $\sin\gamma$ and γ.

The hourly values of $I_\gamma \sin\gamma$ for the average day were now used to determine hourly values of I_γ. These were smoothed, using the requirement that values of I_β should not increase between successive hourly steps of (t) (the time from local noon).

By treating all hourly data for all Aprils, it was possible to obtain hourly values of diffuse radiation on a horizontal surface for the average day (the diffuse component of (S)).

With I_γ, $\sin\gamma$ now known for each hour and with $f'(n/N)$ indicated by Robinson, simple algebra leads to hourly values for D_γ.

5. Results

Cambridge
April

$\phi = 52^\circ\ 13'$
$n = 5.11$ hours, $N = 13.78$ hours, $(n/N) = .371$, $f'(n/N) = .23$
δ on 13th $= 8^\circ\ 46$

Hour identified by	γ	SINγ	I_γ	$I_\gamma \sin\gamma$	$D_{\gamma n}$
t = 7.5°	46.64°	.7270	52	37.80	26.3
22.5	43.26	.6853	51	34.95	24.6
37.5	37.30	.6059	51	30.90	21.0
52.5	29.53	.4929	51	25.14	16.5
67.5	20.75	.3546	51	18.08	10.2
82.5	11.75	.2003	51	10.21	5.4
97.5	02.34	.0407	50	2.03	1.7

Mean daily values

Total radiation on a horizontal surface 318 mw hr cm^{-2}
Diffuse radiation on a horizontal surface 200
Direct radiation on a horizontal surface 118

May n = 4.56 hours, N = 15.58 hours, (n/N) = .456, f'(n/N) = .23
δ on 13th = $18^{\circ}11'$

Hour identified by	γ	SIN γ	I_{γ}	$I_{\gamma}\sin\gamma$	$D_{\gamma A}$
t = 7.5°	55.43°	.8234	53	46.64	29.2
22.5	51.60	.7837	53	41.53	27.4
37.5	45.19	.7082	53	37.53	25.3
52.5	36.93	.6008	53	31.84	20.5
67.5	27.99	.4693	53	24.87	15.7
82.5	18.82	.3226	50	16.13	10.3
97.5	9.84	.1708	50	8.54	5.0
112.5	1.38	.0241	40	0.96	1.5

Mean daily values Total radiation on a horizontal surface 446 mw hr cm^{-2}
Diffuse radiation on a horizontal surface 259
Direct radiation on a horizontal surface 187

June n = 6.72 hours N = 16.72 hours, (n/N) = .403, f'(n/N) = .22
δ on 13th = $23^{\circ}10'$

Hour identified by	γ	$\sin\gamma$	I_{γ}	$I_{\gamma}\sin\gamma$	$D_{\gamma A}$
t = 7.5°	60.38°	.8693	71.0	61.7	23.0
22.5	56.17	.8309	71.0	59.0	22.2
37.5	49.27	.7578	70.8	53.6	19.9
52.5	40.83	.6538	70.6	46.1	16.8
67.5	31.80	.5265	66.5	35.0	14.4
82.5	22.60	.3844	60.0	23.1	10.0
97.5	13.71	.2374	54.0	12.8	6.2
112.5	5.88	.0953	34.0	3.2	2.9

Mean daily values Total radiation on a horizontal surface 511 mw hr cm^{-2}
Diffuse radiation on a horizontal surface 274
Direct radiation on a horizontal surface 237

July n = 6.01 hours, N = 16.38 hours, (n/N) = .367, f'(n/N) = .23
δ on July 13th = 21°57'

Hour identified by	γ	sin γ	I_γ	$I_\gamma \sin\gamma$	$D_{\gamma n}$
t = 7.5°	59.17	.8587	55	47.22	30.6
22.5	54.10	.8100	54	43.74	28.0
37.5	48.26	.7462	54	40.29	26.0
52.5	39.89	.6413	53	33.98	22.3
67.5	30.85	.5128	53	27.07	15.9
82.5	21.69	.3695	51	18.84	11.0
97.5	11.79	.2213	43	9.51	5.9
112.5	4.47	.0780	37	2.89	1.7

Mean daily values Total radiation on a horizontal surface 437 mw hr cm^{-2}
Diffuse radiation on a horizontal surface 273
Direct radiation on a horizontal surface 164

August n = 5.71 hours, N = 14.80 hours, (n/N) = .386, f'(n/N) = .23
δ on August 13th = 14° 45'

Hour identified by	γ	SIN γ	I_γ	$I_\gamma \sin\gamma$	$D_{\gamma n}$
t = 7.5°	52.06°	.7887	57	44.96	24.4
22.5	48.44	.7483	55	41.15	24.3
37.5	42.17	.6713	55	36.92	20.8
52.5	34.20	.5620	55	30.91	17.4
67.5	25.34	.4820	54	23.11	12.0
82.5	16.20	.2790	52	14.50	7.3
97.5	7.12	.1240	45	5.58	2.6

Mean daily values Total radiation on a horizontal surface = 377 mw hr cm^{-2}
Diffuse radiation on a horizontal surface = 225
Direct radiation on a horizontal surface = 152

September n = 4.61 hours, N = 12.81 hours, (n/N) = .36, f' (n/N) = .23
δ on September 13th = 4°06'

Hour identified by	γ	Sinγ	I_γ	$I_\gamma \sin\gamma$	$D_{\gamma A}$
t = 7.5°	41.48°	.6623	67	44.37	17.7
22.5	38.37	.6207	66	40.96	16.5
37.5	33.79	.5413	65	33.45	14.5
52.5	25.37	.4285	64	27.42	10.4
67.5	16.83	.2904	63	18.29	6.5
82.5	7.83	.1362	58	7.90	2.6

Mean daily values Total radiation on a horizontal surface 284 mw-hr cm^{-2}
Diffuse radiation on a horizontal surface 160
Direct radiation on a horizontal surface 124

Meteorological Office
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